SPECIFIC FEATURES OF COMPACTION OF A VISCOPLASTIC MEDIUM WITH A VARIABLE YIELD POINT

L. M. Buchatskii

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An analysis is made of specific features of the viscoplastic behavior for porous materials whose yield point may vary in isothermal conditions, for example, because of structural transformations in the material of the skeleton of the porous body. Conditions of the development of viscous compaction and the compaction kinetics in relation to various conditions are ascertained. The wave propagation of the viscous flow boundary in a specimen with variable porosity is described.

In various technological SHS processes, combined with a pressure treatment of products (in rheosynthesis processes), the time between the combustion termination and the pressure application for different reasons may attain a significant value (of the order of minutes), during which the product temperature falls. As a result, the material transfers from the region of a "hot" deformation to that of a "warm" deformation, where the ability for flow is still retained but the deformation commences on overcoming a certain threshold σ_0, referred to as a yield point. Its magnitude is governed by the material structure and is a function of the temperature. Both these factors may change during the process, but the present study involves the task of analyzing the main qualitative regularities of variation in compaction modes for the SHS materials, which are introduced by the processes of structural transformations not completed by the moment of pressure application.

The temperature dependence of the yield point is quite similar to the exponential viscosity decrease with rising temperature [1]. The dependence on the structural factor is, apparently, difficult to define in the general form in connection with a wide range of the types of structural transformations and with their ambiguous impact on the yield point. Some of the structural transformations (for example, grain growth and coagulation) can lead to an increase in the yield point, whereas others (a variation in the grain shape and a grain ordering) can cause its decrease.

Of prime interest for practical purposes is the case when the yield point grows during the structural transformations, and precisely this circumstance may impede the material compaction. Decreasing the yield point for achieving a porousless state is the most favorable process and, from the rheological standpoint, corresponds to a reverse transition of the material from the region of a "warm" deformation to that of a "hot" deformation.

To quantitatively analyze the effect of the structural factor on the yield point rise and on the compaction modes for the SHS materials, let us select the following form of the dependence of the yield point on the average grain dimension d:

$$\sigma_0 = \sigma_{0\text{M}} \exp\left(-d_{\text{M}}/d\right). \tag{1}$$

The chosen form of the dependence qualitatively accounts for the relationship of the yield point to the structural factor, i.e., at small grain dimensions ($d \to 0$), $\sigma_0 \to 0$ because a limiting decrease in the dimension results in a nonthreshold deformation. At large grain dimensions, the processes occurring at the grain edges, their mutual arrangement, slippage, and joint deformation are no longer of considerable importance; therefore, the yield point tends to its limiting value $\sigma_{0\text{M}}$.

The grain dimension and number alter in the course of structural transformations, which is a prerequisite for a variation in the average grain dimension. Generally speaking, the variation kinetics must be determined from an independent equation, taking into account, wherever possible, all processes of structural change and their influence on the average grain dimension, but in some cases a form of the solution to this equation may be predicted with good assurance. For instance, under the assumption that the grain growth is controlled only by diffusion, we can adopt in the first approximation [2]

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$$d/d_M = kt^\nu, \quad \sigma_0 = \sigma_{0M}\exp(-1/kt^\nu), \tag{2}$$

whereas the time dependence of the yield point is of s-shaped character, with a section of a rapid growth and subsequent saturation. The current study will further utilize the presumed behavior of the yield point during structural transformations.

The rheological equation of state for a nonlinear viscoplastic porous body will be applied, just as in [3-5], in the form

$$\sigma_{ij} = \left(2\eta + \frac{\sigma_0}{W}\right)(e_{ij} - e\delta_{ij}) + \left(\zeta + \frac{\sigma_0}{W}\right)e\delta_{ij} \tag{3}$$

with the following condition of the viscous flow onset:

$$\sigma_* > \sigma_0. \tag{4}$$

The present study employs the designations traditional for problems of hot pressing, viz., η and ζ are the shear and dilatational viscosities of a porous body, η_0 and σ_0 are the shear viscosity and the yield point of an incompressible base; φ and ψ are functions of the porosity Π; σ_{ij} and e_{ij} are the components of the stress and deformation rate tensors; and σ_* and W are the equivalent stress and the deformation rate:

$$\sigma_* = \frac{1}{\sqrt{1-\Pi}}\sqrt{\frac{p^2}{\varphi} + \frac{\tau^2}{\varphi}}\ ; \quad W = \frac{1}{\sqrt{1-\Pi}}\sqrt{\varphi e^2 + \psi\gamma^2}, \tag{5}$$

where p, τ, e, and γ denote, correspondingly, the first and the second invariants of the stress and deformation rate tensors.

In the absence of flow (e = γ = 0), the right side of the rheological relation does not go to zero; rather, it represents physically an effective yield point of the porous body, which is not invariant any longer but depends on the porosity and the form of a stress–strain state. Thus, for example, in the case of plane deformation ($\sigma_r = \sigma_\theta$), the following relations [4] are fulfilled for the tensor invariants σ_{ij} and e_{ij}:

$$\begin{gathered} \eta = \frac{1}{2}\eta_0\varphi, \quad \zeta = \eta_0\psi, \\ \tau = \sqrt{\frac{2}{3}}(\sigma_r - \sigma_z), \quad p = (\sigma_z + 2\sigma_r)/3, \\ \gamma = \sqrt{\frac{2}{3}}(e_r - e_z), \quad e = e_z + 2e_r. \end{gathered} \tag{6}$$

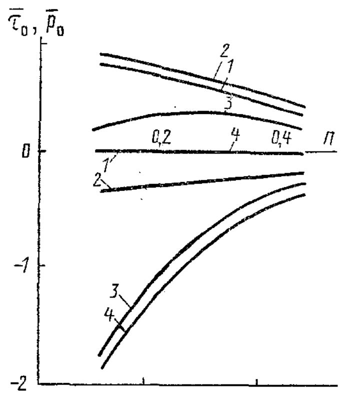
The main components of the tensor of the effective yield point take on the form

$$\begin{gathered} \sigma_0^z = \sigma_0\left(\varphi s - \sqrt{\frac{2}{3}}\psi\right)\sqrt{(1-\Pi)/(\varphi s^2 + \psi)}, \\ \sigma_0^r = \sigma_0(\varphi s + \psi/\sqrt{6})\sqrt{(1-\Pi)/(\varphi s^2 + \psi)} \end{gathered} \tag{7}$$

and we may write their expressions for a simple shear (s = 0), a free upset (s = $-\varphi/\sqrt{6}\psi$)), a molding (s = $-\sqrt{1.5}$), and a uniform compression (s = $-\infty$).

Figure 1 gives ratios of the effective yield points for shear τ_0 and for compression p_0 to the yield point of the incompressible base for each of the listed types of the stress state, depending on the porosity. We may note that, throughout the range of practical initial porosity of 10-50%, the yield point for shear is smaller than the yield point of the incompressible base, whereas the yield point for compression at a porosity smaller than 20% exceeds that for the incompressible base.

Here, it is clearly manifested that the yield point for a porous body depends on the porosity and the type of stress state rather than being an invariant rheological property. The pressure required for the onset of viscous flow in a porous medium may be both lower and higher than the yield point of the incompressible base. Thus, for instance, for an initial porosity of 50%, about 20% of the value of σ_0 is needed, whereas for a porosity of 10%, the pressure must not exceed the yield point of the incompressible base less than twice.



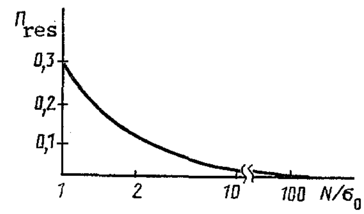


Fig. 1 Fig. 2

Fig. 1. Effective yield points for shear $\bar{\tau}_0 = \tau_0/\sigma_0 \geq 0$ and for compression $\bar{p}_0 = p_0/\sigma_0 \leq 0$ of a porous body vs porosity at various types of a stress state.

Fig. 2. Residual porosity vs the ratio of the applied pressure to the yield point of the skeleton of a porous body.

In the practice of pressing by hydraulic or pneumatic presses, the pressure set by an operator is not gained immediately but rather after some time, depending on the structural characteristics of the press. The compaction of the viscoplastic body also does not begin immediately after the time of pressure application, but when the yield point has been exceeded. However, when the compaction has commenced, the porosity decreases and the effective yield point increases. Hence we may deduce that the time of holding the material under pressure and the mode of variation in this pressure in a working cylinder of the press play an important part in pressing the viscoplastic media. The time-invariable pressure must eventually be counterbalanced by the increased effective yield point in the porous material, and the rapid viscous compaction will terminate, leaving a certain residual porosity in the material. The residual porosity may disappear also due to a purely plastic flow; however, this requires a long-term holding under pressure.

Next let us analyze the most commonly used case of pressing, i.e., that in a rigid cylindrical mold, taking no account of friction on lateral sides. A system of equations, intended for the analysis, with respect to the variable $\rho = 1 - \Pi$ appears as

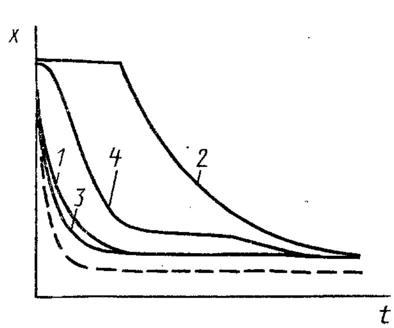
$$\begin{gathered}
\frac{d\rho}{dt_*} = -\rho\frac{dv}{dz}, \quad \frac{d\sigma_z}{dz} = 0, \\
\sigma_z = \left(+\frac{2}{3}\psi + \varphi\right)\left(\eta_* + \frac{\sigma_0}{W}\right)\frac{\partial v}{\partial z}, \\
\sigma_r = \sigma_\theta = \left(-\frac{1}{3}\psi + \varphi\right)\left(\eta_* + \frac{\sigma_0}{W}\right)\frac{\partial v}{\partial z}, \\
\psi = \rho^m; \quad \varphi = \frac{2}{3}\rho^{m-1}/(1-\rho), \quad \sigma_* = \eta_* W^{n-1}, \\
t_* = 0:\ \rho = \rho_0,\ H = H_0, \\
z = 0:\ v = 0, \\
z = H:\ \sigma_z = -N,
\end{gathered} \tag{8}$$

where z and v are the coordinate and the velocity of the material movement, and N is the pressure applied to the upper end of the billet, hereinafter assumed constant or increasing linearly.

For calculations, it is convenient to bring the equations to dimensionless form by using the following variables:

$$x = z/H_0, \quad t = t_*(N_*/\eta_0)^{1/n}, \quad Q_0 = \sigma_0/N_*, \quad Q = N/N_*,$$

$$u = (N_*/\eta_0)^{1/n} v/H_0, \quad e = \frac{\partial v}{\partial z}, \quad \gamma = -\sqrt{\frac{2}{3}}\frac{\partial v}{\partial z}, \quad q = \frac{2\rho^m}{3(1-\rho)}.$$



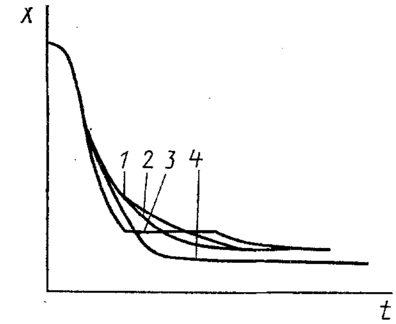


Fig. 3 Fig. 4

Fig. 3. Compaction kinetics of a viscoplastic porous body at various rates of increase in the applied pressure and in the yield point.

Fig. 4. Compaction kinetics of a porous body at various degrees of nonlinearity of the rheological behavior.

Having done the necessary substitutions and manipulations, we arrive at the following form of the kinetic equation for a relative density ρ:

$$\frac{d\rho}{dt} = \left(\frac{Q}{\sqrt{q\rho}} - Q_0\right)^{1/n} \frac{1}{\rho} \sqrt{\frac{q}{\rho}}, \quad q = \frac{2\rho^m}{3(1-\rho)}, \tag{9}$$

whence some qualitative and quantitative inferences can be made. Unlike the case of compaction of a viscous porous body [6], there is a prerequisite for the derivative $d\rho/dt$ going to zero:

$$\frac{2}{3} \rho_{\min}^{m+1} \left(\frac{Q}{Q_0}\right)^2 + \rho_{\min} = 1, \quad \frac{Q}{Q_0} = \frac{N}{\sigma_0}. \tag{10}$$

Hence the minimal porosity, at which the viscous flow ends, may be determined. Obviously, when the yield point is absent ($Q_0 = 0$), the compaction terminates only at a nonzero porosity. Any nonzero value of Q_0 gives rise to the residual porosity, the greater the more the applied pressure exceeds the yield point. Figure 2 plots the residual porosity vs the indicated relation for m = 1. Clearly, about a tenfold excess over the level of the yield point for the incompressible base results in the compaction actually to a porousless state. Running ahead, it should be noted that all results of the current study are very weakly dependent on the parameter m.

Figure 3 presents predictions for the compaction kinetics of a viscoplastic body in comparison with the compaction kinetics of a linear-viscous body (a dashed line).

Curve 1 corresponds to the compaction of the material with a constant yield point σ_0 = const under the action of an unchangeable ambient pressure ($N/\sigma_0 = 2.5$). Since the applied pressure is much in excess of the yield point σ_0, the compaction begins immediately from the instant of pressure application (t = 0) and terminates on the residual porosity attaining $\Pi_{res} = 8.9\%$, at which the effective yield point is compared with the pressure applied from without.

If the applied pressure increases in time linearly from zero to an assigned value ($N/\sigma_0 = 2.5$), as curve 2 shows, then there is no compaction at the initial instants of time. The porous billet compaction starts only after condition (4) has been fulfilled. The process ends with attaining the same residual porosity as in the previous case, because the values of N and σ_0 remained unchanged. If the pressure is allowed to rise still more by increasing the ratio N/σ_0, the residual porosity will decrease.

The compaction of the material with a variable yield point by the action of constant pressure (curve 3) differs from the previous cases by a compaction deceleration, since during the compaction the yield point of the incompressible base increases from 0 up to $N/\sigma_0 = 2.5$.

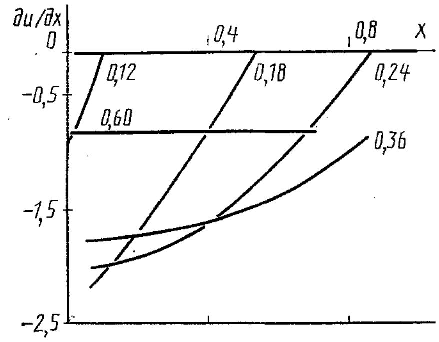


Fig. 5. Propagation of a viscous compaction wave in a viscoplastic material with a nonuniform porosity distribution.

The case is most diverse when both the yield point and the applied pressure alter simultaneously but at different rates (curve 4). At first, the compaction commences even under the effect of low pressure, because the initial period of the yield point growth is longer than that for the pressure. After a certain "induction" period the yield point sharply increases, violating condition (4), i.e., the compaction stops and a step forms on the curve. Meanwhile, the competitive processes of the pressure and yield point growth persist, and relationship (4) changes once again in favor of continuation of the compaction, which ceases already as a consequence of the rise in the effective yield point rather than in σ_0.

Figure 4 illustrates the effect of a nonlinear dependence of rheological properties on the porosity (the parameter m) and on the deformation parameters (the parameter n) in comparison with a linear dependence on the above-mentioned parameters. As was already pointed out, the value of m affects the results slightly. Its threefold increase (curve 2) leads only to some acceleration of the compaction; the attainment of the maximal porosity needs a time shorter by 10%. A threefold increase in the degree of rheological nonlinearity ($n = 3$) qualitatively changes the compaction pattern: from a monotonic decrease in the compaction rate (curve 1) we obtain step-like curve 3, with a section where there is no viscous compaction whatsoever.

Curve 4 shows the influence of the rate of the yield point growth on the kinetics of the material compaction. As compared with curve 1, the growth rate considered is reduced to one-third; therefore, having one and the same ratio N/σ_0, by virtue of a retarded yield point rise we obtain in the latter case a pattern more resembling the compaction by constant pressure, higher than the yield point. The smaller residual porosity is attributed to the shortage of the process time, during which the yield point σ_0 failed to reach its limiting value.

All the aforesaid pertains to a uniform deformation when all characteristics inside the porous body are distributed uniformly. However, with a nonuniform porosity distribution, often encountered in practice, the distribution of the pressing properties and characteristics over the specimen is not uniform either.

Figure 5, which gives calculated profiles of the velocity gradient for the specimen with a nonuniform porosity distribution at various instants of time (numerals by the curves), can serve as an illustration of the propagation of a viscous compaction wave in a viscoplastic porous medium. We recall that the velocity gradient in the case of pressing considered is, within a sign, a compaction rate; therefore it is convenient to study for analyzing the propagation of viscous flow boundary.

As a case in point, we selected a linear porosity increase from the upper ($x = 1$) to the lower ($x = 0$) edge of the billet: $\Pi = 0.4 + 0.2x$. At the initial instant of time ($t = 0$), there is no compaction and a zero velocity gradient is observed over the entire volume. Within the time (dimensionless) $t = 0.12$ of applying the pressure $N/\sigma_0 = 2.5$, the viscous flow and the associated porosity variation have spread only over a small bottom region of the material ($x < 0.2$) with the greatest porosity, whereas the remaining part keeps on behaving as an entity. After a lapse of $t = 0.18$, the flow has already spread over half the volume, whereas the upper part behaves as an incompressible body. Within $t = 0.24$, only the uppermost layers remain free from the viscous flow. Within $t = 0.36$, the entire volume is subjected to the viscous compaction, and a porosity equalization is observed in the lower layers. The total height of the specimen is reduced to 0.82. The subsequent compaction involves the effect of density self-equalization [5], which leads to smoothing of the velocity gradient profile, and the porosity reduction entails a decrease in its absolute value.

Based on the results of the analysis, we may distinguish the following salient features of the compaction of a nonlinear viscoplastic medium:

1. The compaction commences after the applied pressure has exceeded the effective yield point. The latter is a tensor, whose components are dependent on the porosity and the type of stress state.

2. The porosity reduction increases the effective yield point, and the compaction by any constant pressure gives rise to a noticeable residual porosity in the material if the ratio of the applied pressure to the yield point for the incompressible base is smaller than 10.

3. Compaction of the material with a yield point varying due to structural transformations by an increasing pressure may bring about steps on the compaction curve. The presence of steps and their size depends on the rates of the pressure and yield point growth and on the rheological nonlinearity.

4. In the billets with a nonuniform porosity distribution, the compaction starts in the microvolumes with a maximal porosity and afterwards propagates in a wave mode to the entire remaining volume. The compaction throughout the volume is characterized by a porosity self-equalization.

NOTATION

σ_0, σ_{0M}, yield point of the material of the porous body skeleton and its value in the case of a monocrystalline state, Pa; d, d_M, average grain dimension and scale factor, m; σ_*, W, equivalent stresses and deformation rate, Pa and 1/sec; η_0, shear viscosity of the incompressible base, Pa·sec; φ, ψ, functions of the porosity Π or of the relative density $\rho = 1 - \Pi$; σ_0^i, p_0, τ_0, components of the tensor of the effective yield point, Pa; t_*, t, dimensional and dimensionless time, sec; z, x, dimensional and dimensionless coordinate, m; H_0, H, initial and running height of the specimen, m; N_*, scale value for stresses, Pa; N, Q, dimensional and dimensionless pressure applied to the specimen end, Pa; m, n, nonlinearity degrees of the dependence of rheological properties on porosity and effective rate of deformation.

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